MMAT 5340 Assignment #1

Please submit your assignment online on Blackboard

Due at 12:00 pm on Wednesday, September 15, 2021

1. Let X be a random variable with uniformly distribution on interval $[-1, 1]$, i.e. its probability density function is given by

$$\rho(x) = \begin{cases} 1/2, & \text{if } x \in [-1, 1], \\ 0, & \text{otherwise.} \end{cases}$$

   Prove first that $Cov(X, X^2) = 0$, and then show that X and X^2 are not independent.

2. Let X be a random variable with Gaussian distribution, whose probability density function is given by (for some constant $\sigma > 0$)

$$\rho(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}, \quad x \in \mathbb{R}.$$

   Compute the following values

   a) $\mathbb{E}[X]$, $\mathbb{E}[X^2]$, $\mathbb{E}[X^3]$.
   b) $\mathbb{E}[\exp(cX)]$, for some constant $c \in \mathbb{R}$.

3. Let X be a discrete random variable with the probability function (for some constant $p \in (0, 1)$)

$$\mathbb{P}(X = k) = (1 - p)p^k, \quad k \in \mathbb{Z}_{\geq 0}.$$

   Compute the following values

   a) $\mathbb{E}[X]$, $\mathbb{E}[X^2]$.
   b) $\mathbb{E}[\exp(cX)]$, for some constant $c \in \mathbb{R}$.